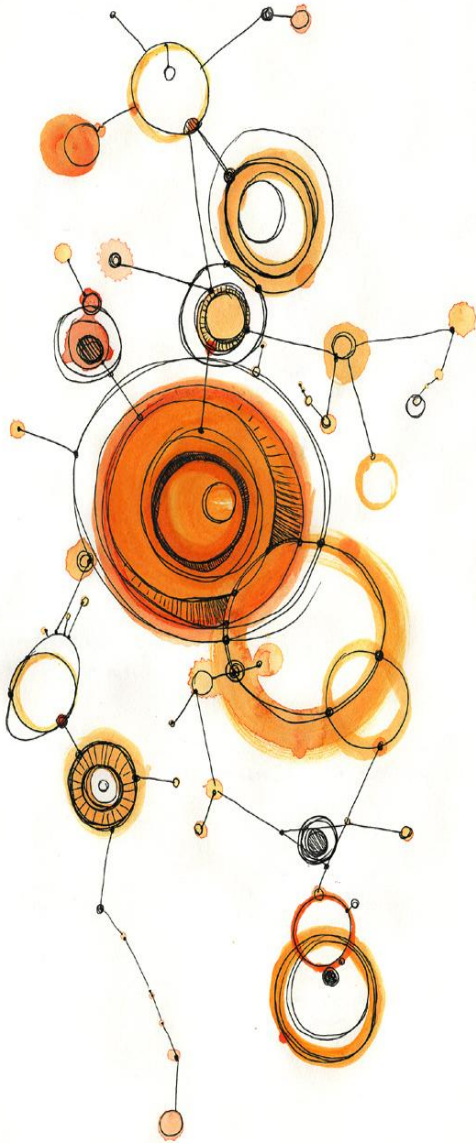


# Getting the Best from Uncertain Data: the Correlated Case

*Ilaria Bartolini, Paolo Ciaccia  
and Marco Patella*

DEIS

University of Bologna  
Italy



# Talk outline

- Preliminaries on
  - skyline query and
  - probabilistic (uncertain) databases
- Skyline on probabilistic relations including *correlation among tuples*
  - Notion of probabilistic domination
  - How to efficiently check probabilistic domination
- Implementation for different ranking semantics
- Time complexity analysis of algorithms for the resolution of skyline queries
- Conclusion and future work

# Skyline of a deterministic relation

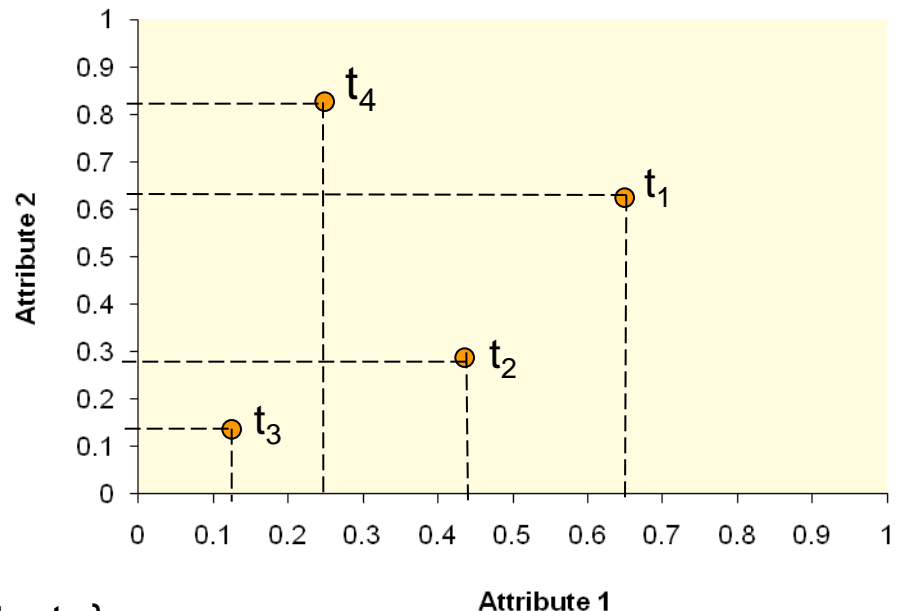
- The Skyline of a deterministic relation  $R$  returns all the objects that are **not dominated** by any other object

“An object dominates another object if it is equal or better in all its dimensions and better in at least one dimension”

$$SKY(R) = \{u \in R \mid \nexists v \in R : v \succ u\}$$

Traffic-monitoring application

TID	Plate No	Time	Speed
$t_1$	H-111	11:20	145
$t_2$	W-266	11:05	137
$t_3$	X-255	10:50	135
$t_4$	C-444	10:55	155



Skyline =  $\{t_1, t_4\}$

# Probabilistic databases

- Uncertain data can be represented through *probabilistic relations*, in which each tuple has also a *probability* (confidence) to appear in an instance of the relation

Traffic-monitoring application (sample of last-hour recording)

TID	Plate No	Radar	Time	Speed	Prob
t1	X-123	L1	10:53	90	0.2
t2	X-246	L2	10:50	100	0.15
t3	X-246	L3	10:40	95	0.1
t4	X-456	L1	10:32	110	0.1
t5	X-456	L2	10:30	130	0.3
t6	X-121	L3	10:30	110	0.2
t7	X-324	L4	10:30	90	0.5
t8	X-827	L4	10:20	105	0.35
t9	X-827	L5	10:15	90	0.4
t10	X-442	L5	10:10	120	0.3
t11	X-442	L2	10:05	140	0.1

- We apply the notion of skyline to the case of probabilistic relations including *correlation among tuples*
  - x-relation model* (i.e., mutual exclusion rules)

Rule: “because radar location, a same car cannot be detected by two radars within an interval of one hour”

→ tuples *t2* and *t3* can not be part of a same instance of the relation, for example

# Contributions and basic properties

- In (*Bartolini et al., SEBD'11*) we have shown how *skyline queries can be defined* for a probabilistic relation  $R_p$ 
  - the case of *independent tuples* was analyzed
- In this paper we extend the applicability of skyline queries to the “*correlated*” case (*x-relations*)
  - Our probabilistic domination definition is general
    - does not depend on the *ranking semantics* used
  - Its implementation depends on the specific *ranking semantics*
  - We detail the analysis for 5 commonly used *ranking semantics*
- A *ranking semantics*, given a linear order of (deterministic) tuples and a probabilistic model, produces a new (probabilistic) ranking of tuples
  - **Different** ranking semantics give with the **same** input **different** probabilistic rankings!

# Skyline of $R_p$ depends on ranking semantics

- It is well known that different ranking semantics yield quite different results for top- $k$  queries in probabilistic databases
- This is also the case when such semantics are used for skyline queries
- Continuing our running example:  
“A skyline query on the **Time** and **Speed** attributes finds those readings that are the most recent ones and concern high-speed cars”
- In the deterministic case, it is:  $SKY(R) = \{ t_1, t_2, t_4, t_5, t_{11} \}$

## Traffic-monitoring application

Ranking semantics	$SKY(R_p)$
Expected Rank	$\{t_5, t_7\}$
Expected Score	$\{t_1, t_2, t_4, t_5, t_7, t_8, t_{11}\}$
U-Top1, U-1Ranks, Global-Top1	$\{t_1, t_5\}$

Although  $t_5$  is part of all considered skylines, this is not the case for other tuples (e.g.,  $t_7$ )

# Basic ingredients

- Domination relation  $\succ$  between tuples  $u$  and  $v$  is a *strict partial order*
- A *linear order*  $\triangleright$  is a strict partial order that is also *connected* (either  $u \triangleright v$  or  $v \triangleright u$ )
- $\triangleright$  is called *linear extension* of  $\succ$  iff  $u \succ v \implies u \triangleright v$
- Any strict partial order  $\succ$  equals the intersection of its linear extensions

$$\succ = \bigcap \{ \triangleright \mid \triangleright \in \text{Ext}(\succ) \}$$

- A *probabilistic ranking function*  $\Psi$  is a function that, given  $R_p$  and a *linear order* on the (deterministic) tuples of  $R$ , yields a *probabilistic linear order*  $\triangleright_p = \Psi(\triangleright, R_p)$  on the probabilistic tuples of  $R_p$
- The actual *ranking* of the probabilistic tuples is obtained by computing for each tuple  $u$  a value  $\psi_{\triangleright}(u)$  so that

$$u \triangleright_p v \iff \psi_{\triangleright}(u) > \psi_{\triangleright}(v)$$

# Let's put it all together

## Probabilistic domination (P-domination for short):

- Given two tuples  $u$  and  $v$  in  $R_p$ , we say that  $u$  P-dominates  $v$  (i.e.,  $u \succ_p v$ )

$$u \succ_p v \Leftrightarrow u \triangleright_p v, \forall \triangleright_p = \psi(\triangleright, R_p), \triangleright \in \text{Ext}(\succ)$$



$$u \succ_p v \Leftrightarrow \psi_{\triangleright}(u) > \psi_{\triangleright}(v), \forall \triangleright \in \text{Ext}(\succ)$$

## SKY( $R_p$ ):

- Consequently, similarly to the deterministic case, the skyline of  $R_p$  is defined as:

$$\text{SKY}(R_p) = \{u \in R \mid \exists v \in R : v \succ_p u\}$$

where the only difference with deterministic case is that  $>$  is replaced with  $\succ_p$



# Our approach

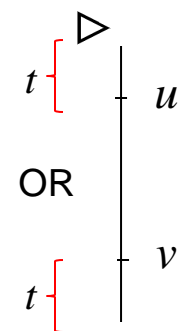
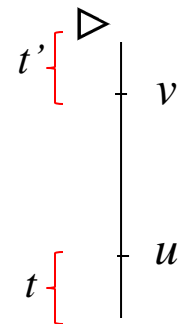
- Goal: check P-domination without materializing any linear extension of  $\succ$
- Note that for  $u \succ_p v$  to hold it has to be  $\psi_{\triangleright}(u) > \psi_{\triangleright}(v)$  for all linear extensions  $\triangleright$  of  $\succ$ , that is:

$$u \succ_p v \Leftrightarrow \min_{\triangleright \in \text{Ext}(\succ)} \left\{ \frac{\psi_{\triangleright}(u)}{\psi_{\triangleright}(v)} \right\} > 1$$

- The key idea is:
  - **if** we find a linear order  $\triangleright$  that is the *most unfavorable one* for  $u$  with respect to  $v$ , and  $\psi_{\triangleright}(u) > \psi_{\triangleright}(v)$  holds for this “extremal” order
  - **then** it will necessarily hold for any other linear extensions of  $\succ$

# How to check P-domination

- When comparing two tuples  $u$  and  $v$ , we analyze how other tuples should be arranged in the linear order so as to minimize the ratio  $\psi_{\triangleright}(u) / \psi_{\triangleright}(v)$
- Two relevant cases (regardless of the specific probabilistic ranking function  $\Psi$ ):
  - $u \not\triangleright v$ : if  $u$  does not dominate  $v$ , the extremal linear order corresponds to the case where:
    1.  $u \triangleright t$  only for those tuples  $t$  that  $u$  dominates, and
    2.  $t' \triangleright v$  only for those tuples  $t'$  that dominate  $v$
  - $u \succ v$ : when  $u$  dominates  $v$ ,  $u \triangleright v$  has necessarily to hold
    - We just have to determine the order of tuples  $t$  which are “indifferent” to both  $u$  and  $v$ 
      - Each of such tuples is necessarily sorted either above  $u$  (so as to unfavor both  $u$  and  $v$ ) or under  $v$  (so as to favor both  $u$  and  $v$ )



# Ranking semantics

- Expected rank (*Cormode et al.*, ICDE'09)
  - $\psi_{\triangleright}(u)$  is the average rank of  $u$
- Expected score (*Cormode et al.*, ICDE'09)
  - $\psi_{\triangleright}(u)$  is the average score of  $u = p(u) \times s(u)$
- U-Top $k$  (*Soliman et al.*, ICDE'07), U- $k$ Ranks (*Soliman et al.*, ICDE'07), Global-Top $k$  (*Zhang et al.*, DBRank'08)
  - We are only interested in top-1 results (Skyline is the union of all top-1 results for any linear order)
  - All ranking semantics give the same result for  $k=1$
  - $\psi_{\triangleright}(u)$  is the probability of  $u$  to be the top-1 tuple

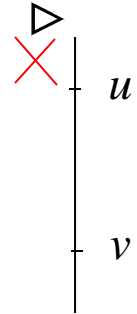
# Expected score

- $\psi_{\triangleright}(u)$  is the average score of  $u = p(u) \times s(u)$ 
  - Does not depend on the tuples that precede  $u$
  - Does not depend on the correlation model

- It is:

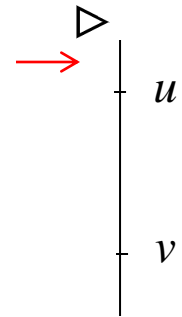
$$u \succ_p v \Leftrightarrow u \succ v \wedge \frac{p(u)}{p(v)} \geq 1$$

- Note that above result is valid for *any* correlation model
  - Not only for x-relations



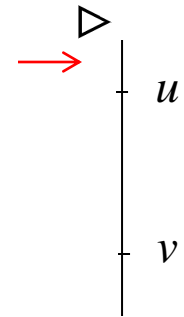
# U-Top1, U-1Ranks, and Global-Top1

- $\psi_{\triangleright}(u)$  is the probability of  $u$  to be the top-1 tuple
  - Thus,  $\psi_{\triangleright}(u)$  *only depends on tuples that precede  $u$*
- For x-relations  $\psi_{\triangleright}(u)$  is computed as a “*product of sums*”
- It follows that a tuple  $t$  that is *indifferent* to both  $u$  and  $v$ :
  - If  $t$  is mutually exclusive to  $v$ , then  $t$  should be ordered above  $u$ 
    - $t$  does not influence the probability of  $v$  to be the top-1 tuple, because there is no instance containing both  $t$  and  $v$
    - We do not favor  $u$
  - Otherwise,  $t$  should be ordered under  $v$ 
    - We do not unfavor  $v$



# Expected rank

- $\psi_{\triangleright}(u)$  is the average rank of  $u$ 
  - Thus,  $\psi_{\triangleright}(u)$  only depends on tuples that precede  $u$
- For x-relations  $\psi_{\triangleright}(u)$  is computed as a “sum of products”
- It follows that a tuple  $t$  that is *indifferent* to both  $u$  and  $v$ :
  - If  $t$  is mutually exclusive to  $v$ , then  $t$  should be ordered above  $u$ 
    - $t$  does not influence the average rank of  $v$ , because there is no instance containing both  $t$  and  $v$
    - We do not favor  $u$
  - If  $t$  is mutually exclusive to  $u$ , then  $t$  should be ordered under  $v$ 
    - $t$  does not influence the average rank of  $u$ , because there is no instance containing both  $t$  and  $u$
    - We do not unfavor  $v$
  - Otherwise,  $t$  could be ordered either above  $u$  or under  $v$ 
    - It depends on the probabilities of  $u$  and  $v$
    - Either we do not favor  $u$  or we do not unfavor  $v$



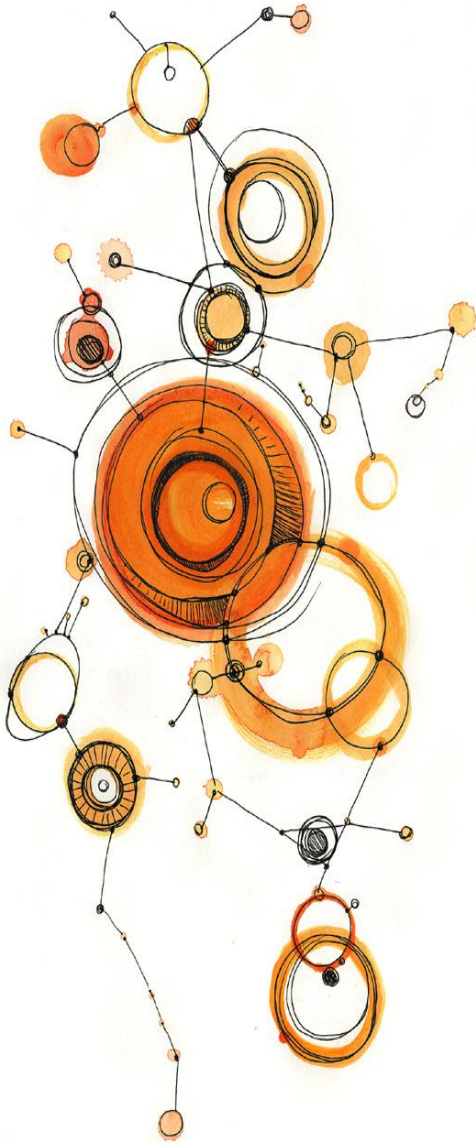
# Complexity analysis

- Time complexity for computing the  $SKY(R_p)$ ,  $R_p$  consists of  $N$  tuples
  - In the worst case we should compare every pair of tuples
- 1. Expected score
  - A single condition with  $O(1)$  complexity
  - Overall complexity  $O(N^2)$
- 2. U-Top1, U-1Ranks, Global-Top1
  - $u \not\asymp v$  with  $O(1)$  complexity (with pre-computation)
  - $u \succ v$  with  $O(N)$  complexity (possibly all tuples are indifferent to each other)
  - Overall complexity  $O(N^3)$
- 3. Expected rank
  - $u \not\asymp v$  with  $O(1)$  complexity (with pre-computation)
  - $u \succ v$  with  $O(N)$  complexity (possibly all tuples are indifferent to each other)
  - Overall complexity  $O(N^3)$
- For 2. and 3., for the case  $u \succ v$ , we propose an  $O(1)$  sufficient condition so as to postpone the  $O(N)$  check as much as possible

# Conclusions and future work

- In this extended abstract we apply the notion of skyline to the case of probabilistic relations
- We elaborated our analysis for the “correlated” case of x-relations, consisting of a set of generation rules specifying the mutual exclusion of tuples
- Extended version of the paper has been accepted for publication on the IEEE TKDE journal
- We argue that there is not a “best” ranking semantics, rather the choice might depend on the specific application at hand and user preferences
  - Understanding the properties of the different semantics is therefore a, both practical and theoretical, interesting research issue
- Moreover, we are also interested in considering other, more expressive, correlation models (e.g., and/xor trees and probabilistic graphical models)





# Thank you!

