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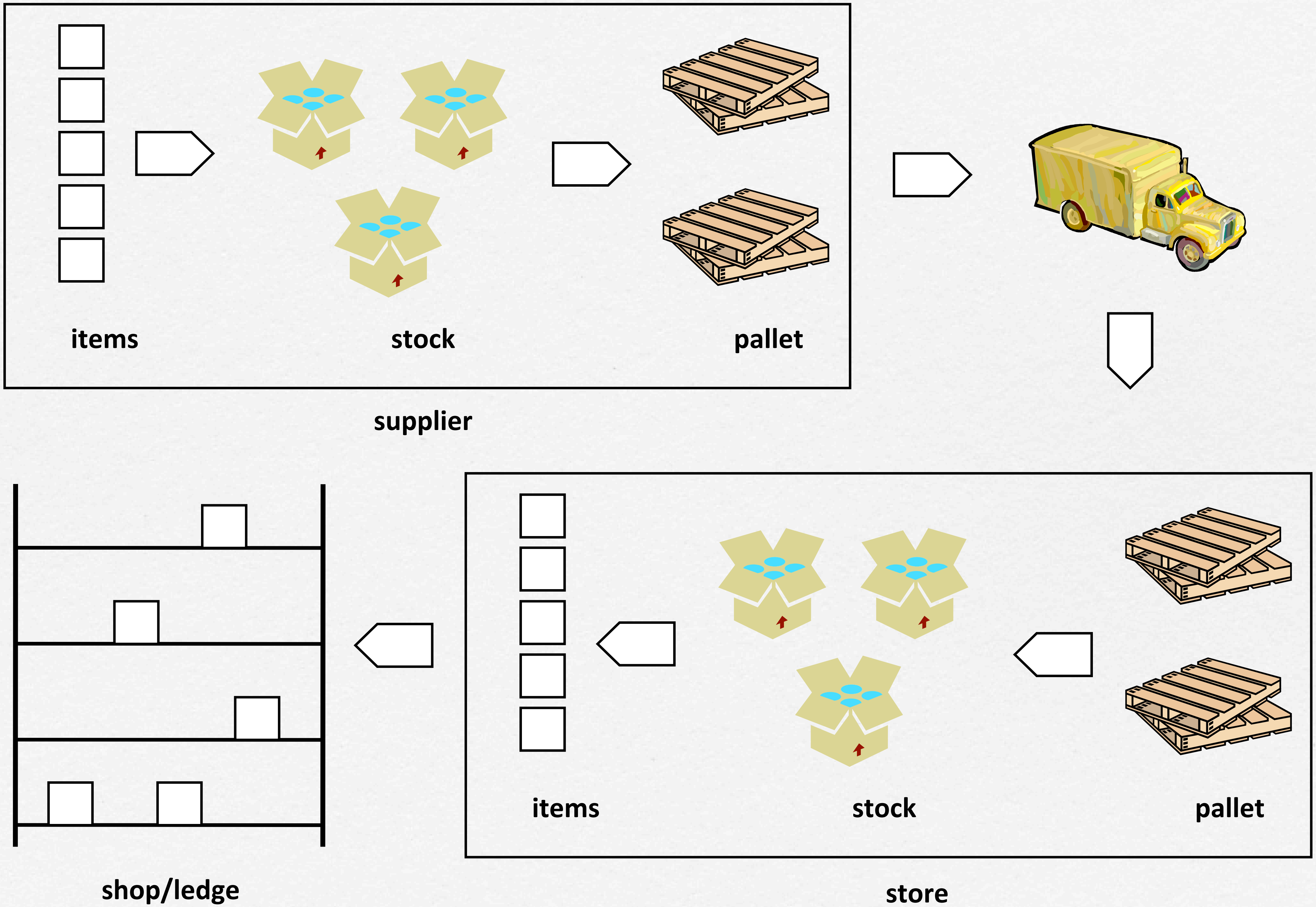
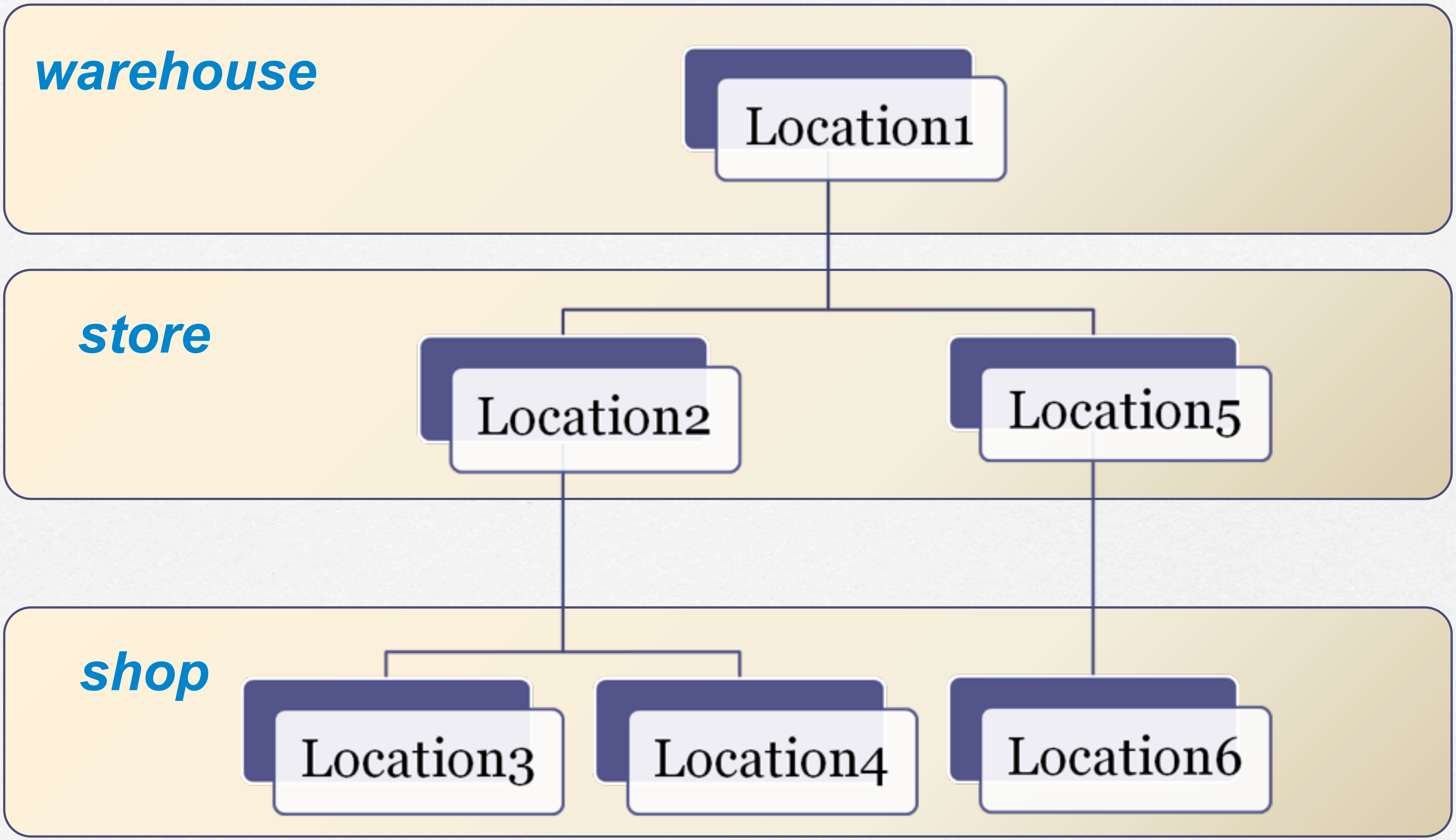
A linear algebra approach for Supply Chain Management

Roberto De Virgilio and Franco Milicchio

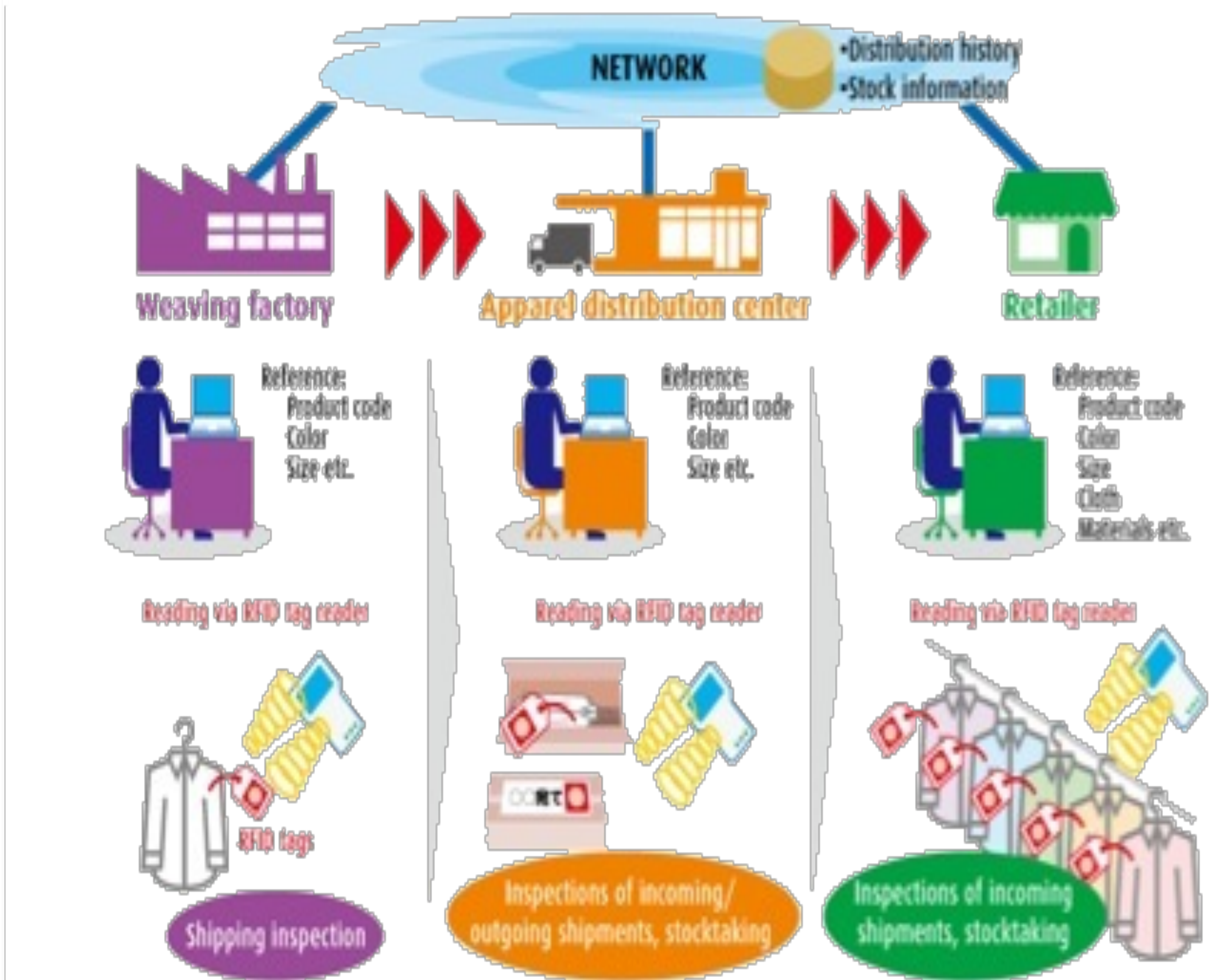
26 June 2012 - Venice

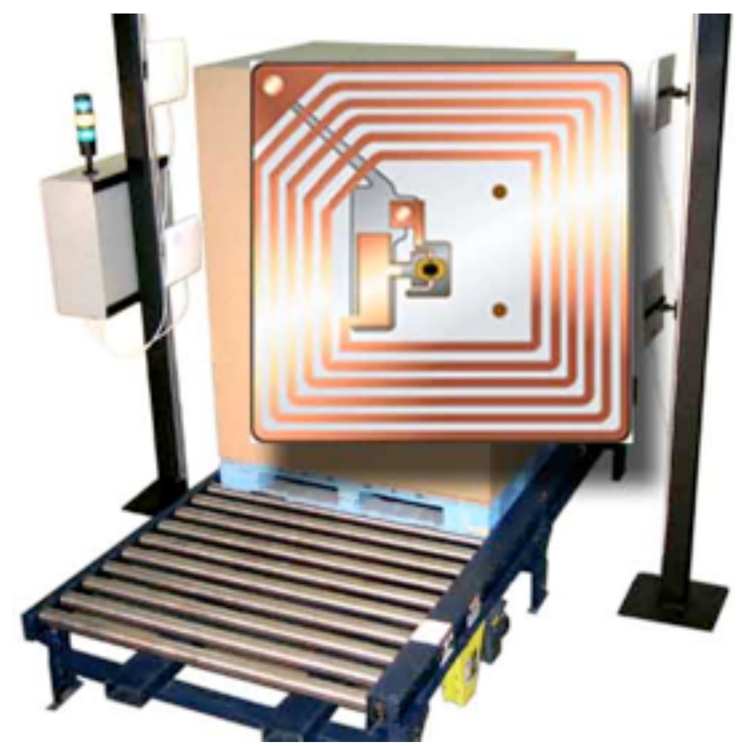
Supply Chain Management



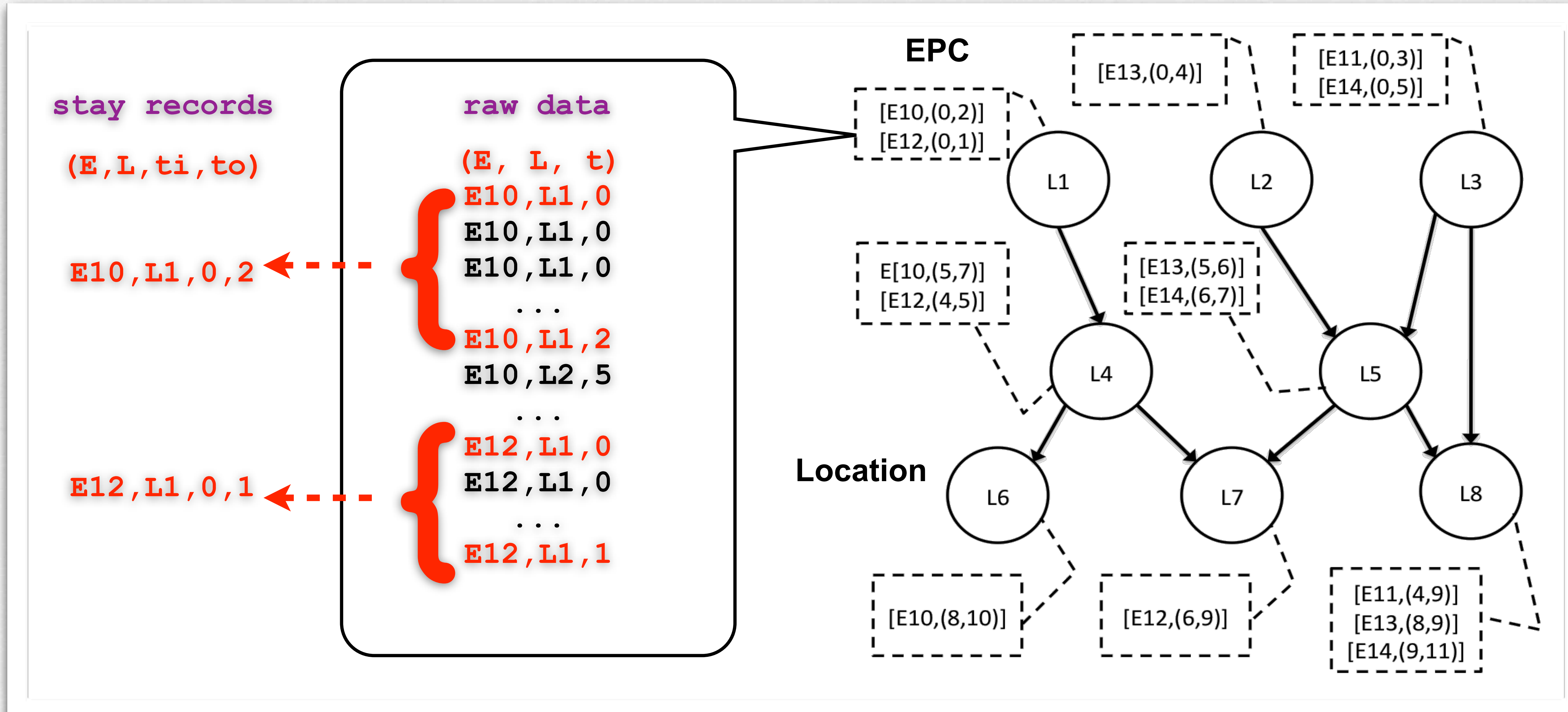

warehouse
Location1
store
Location2
Location5
shop
Location3
Location4
Location6

Radio Frequency IDentification (RFID)

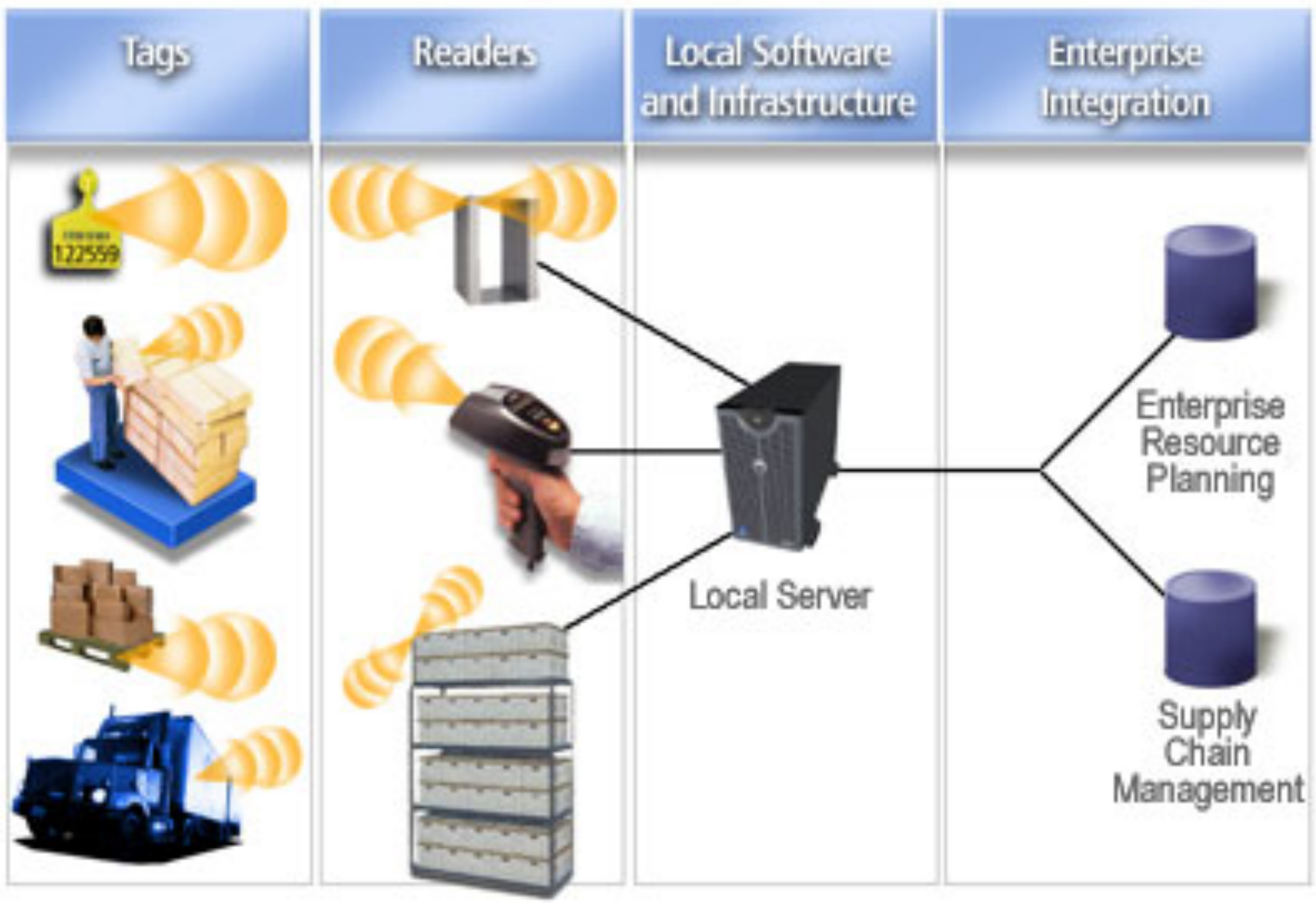




RFID data *modeling*



RFID Data Management



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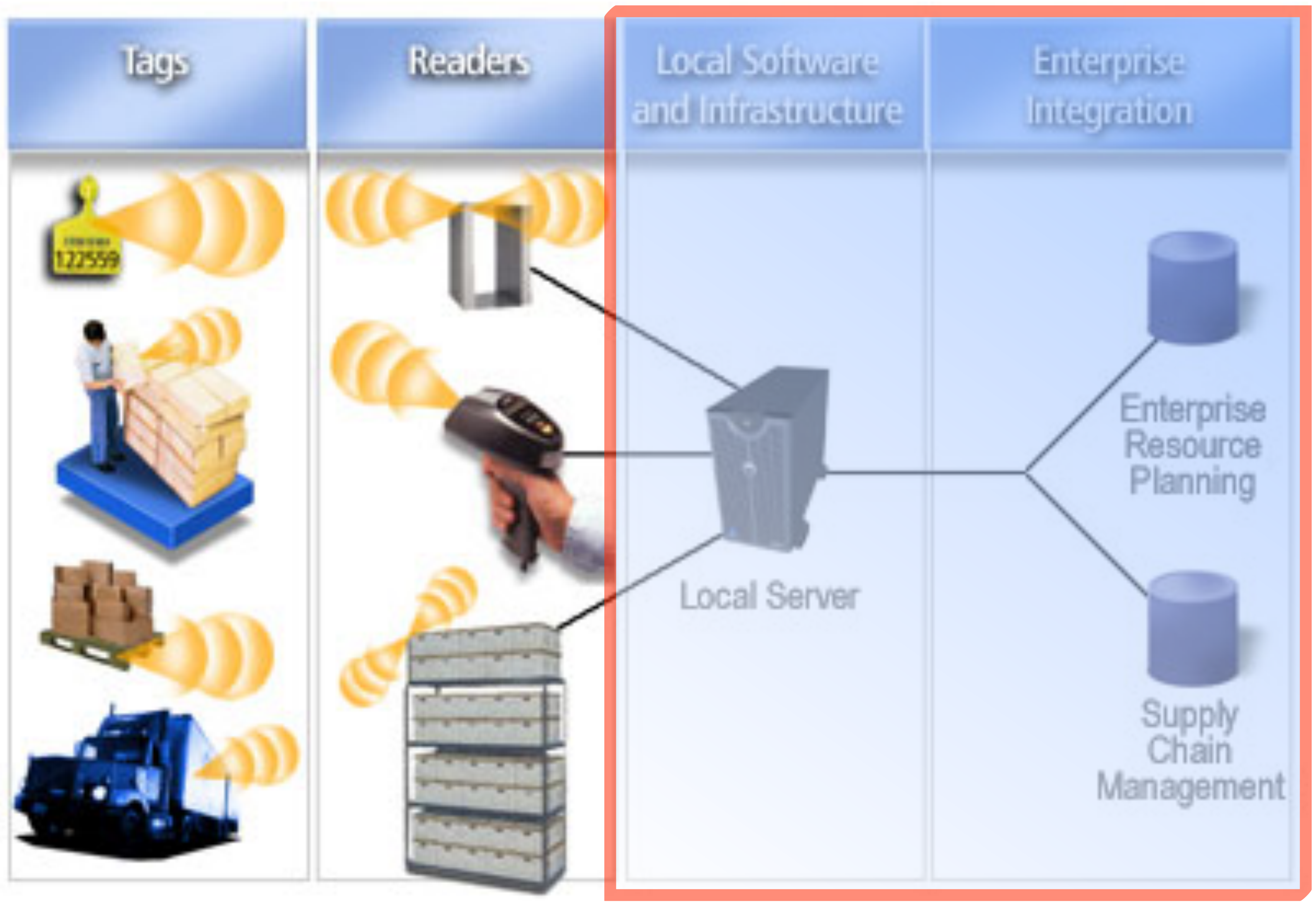
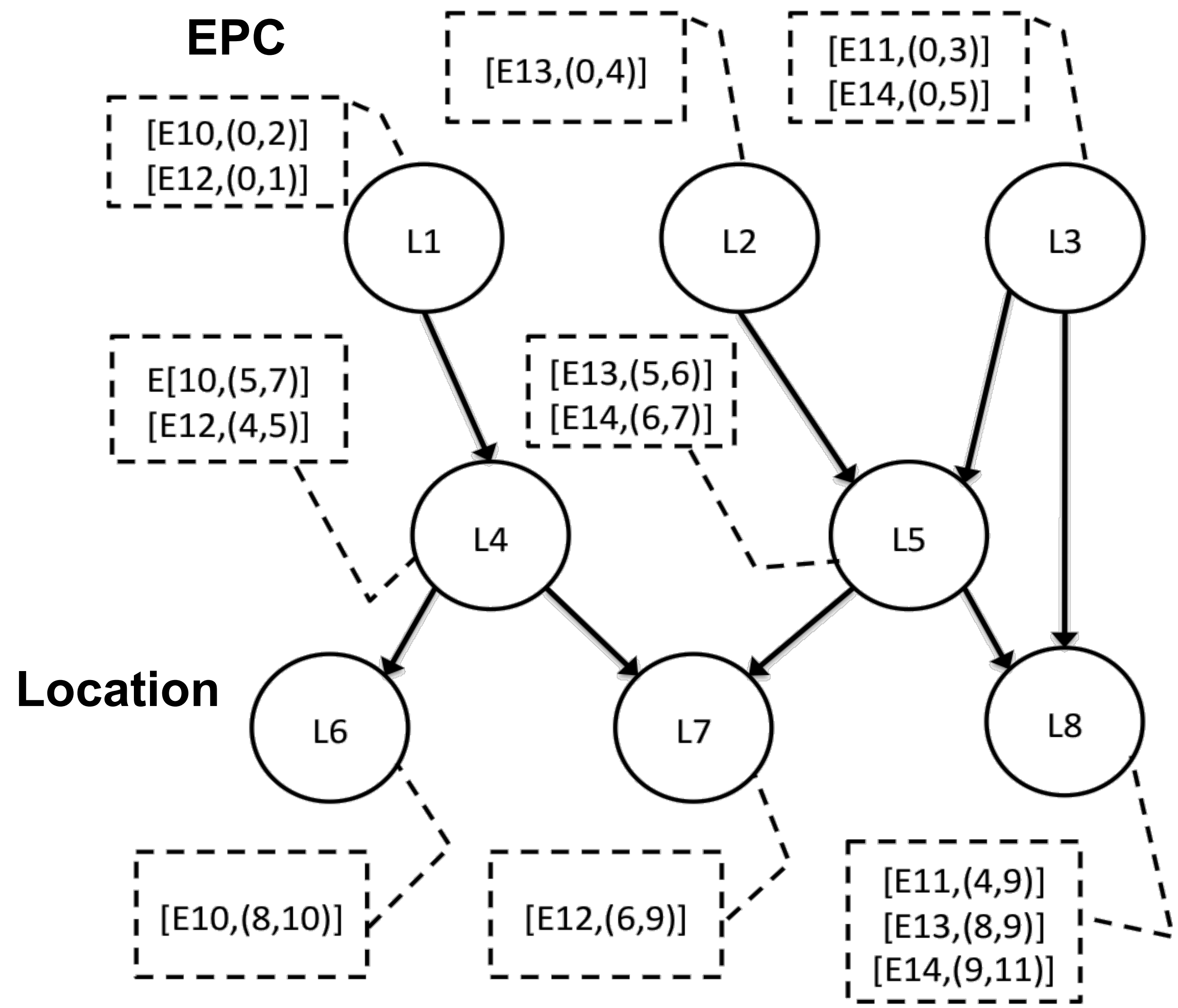


RFID data *analysis*

- **Tracking Query**: es. E10



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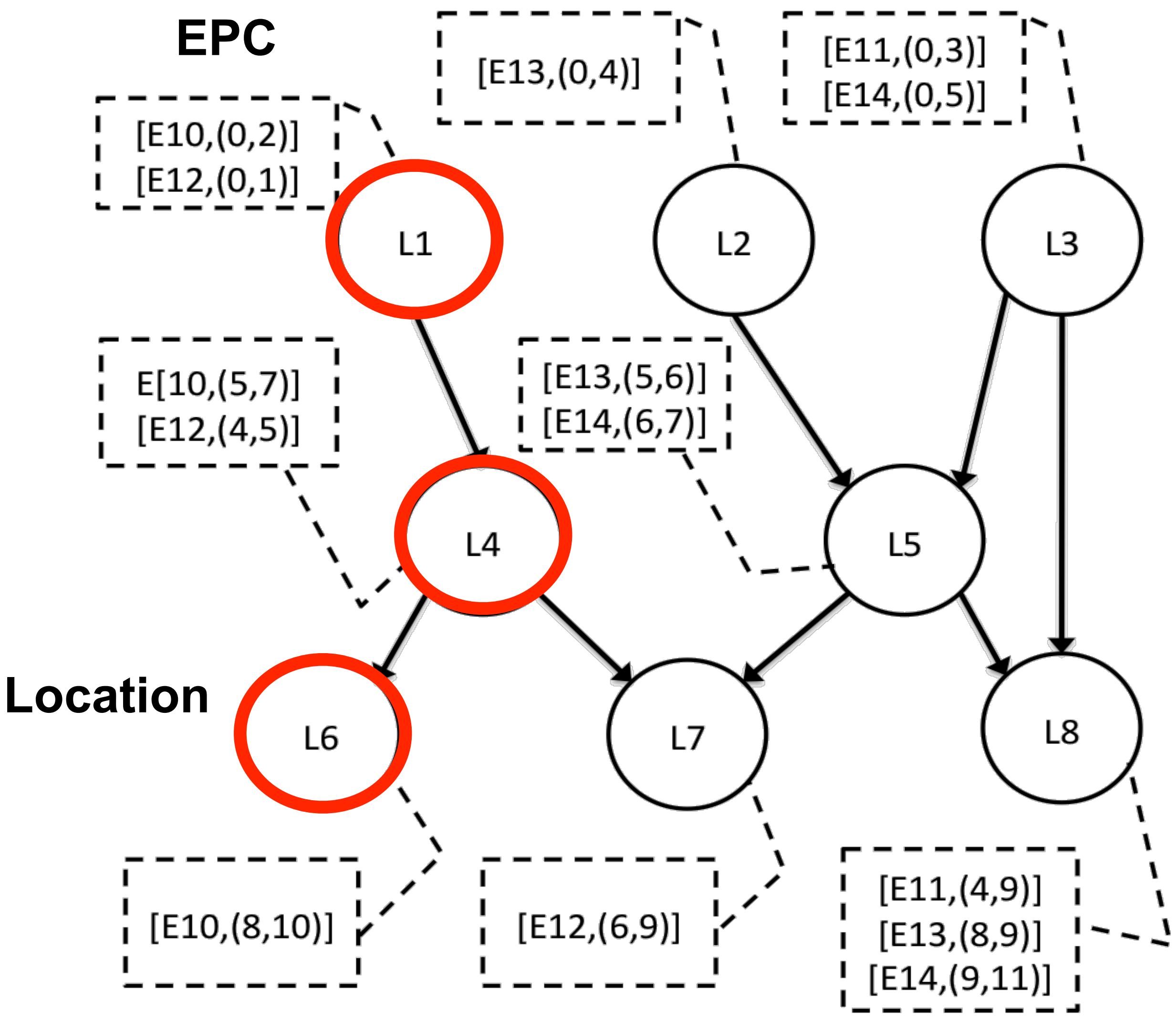


RFID data *analysis*

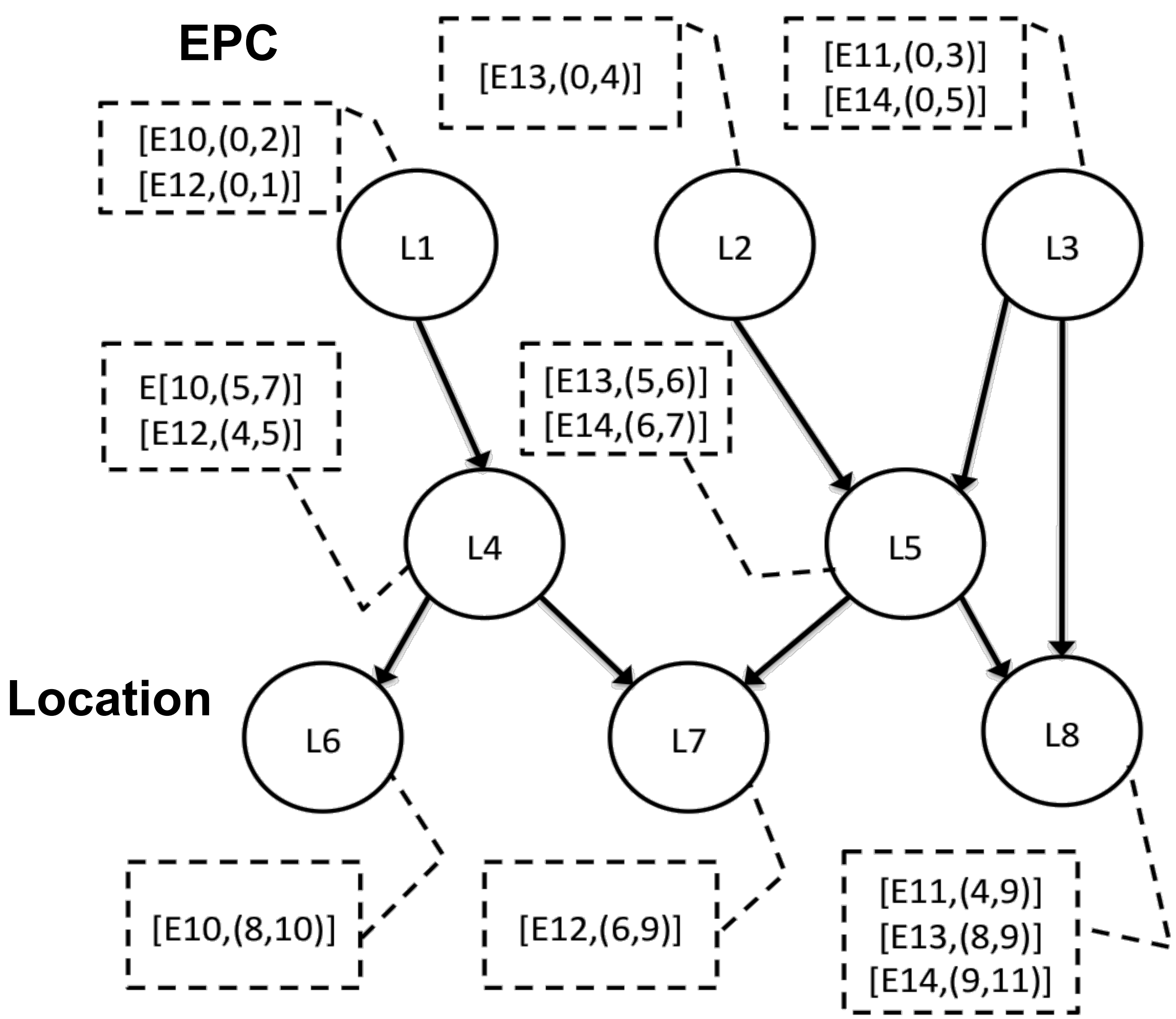
- **Path Tracking: es.** $L3[StartTime > 0]//L8[EndTime - StartTime < 4]$.

$$L_1[\text{cond}_1]//L_2[\text{cond}_2]//\ldots//L_n[\text{cond}_n]$$

or

$$L_1[\text{cond}_1]/L_2[\text{cond}_2]/\ldots/L_n[\text{cond}_n]$$

SOL: E14



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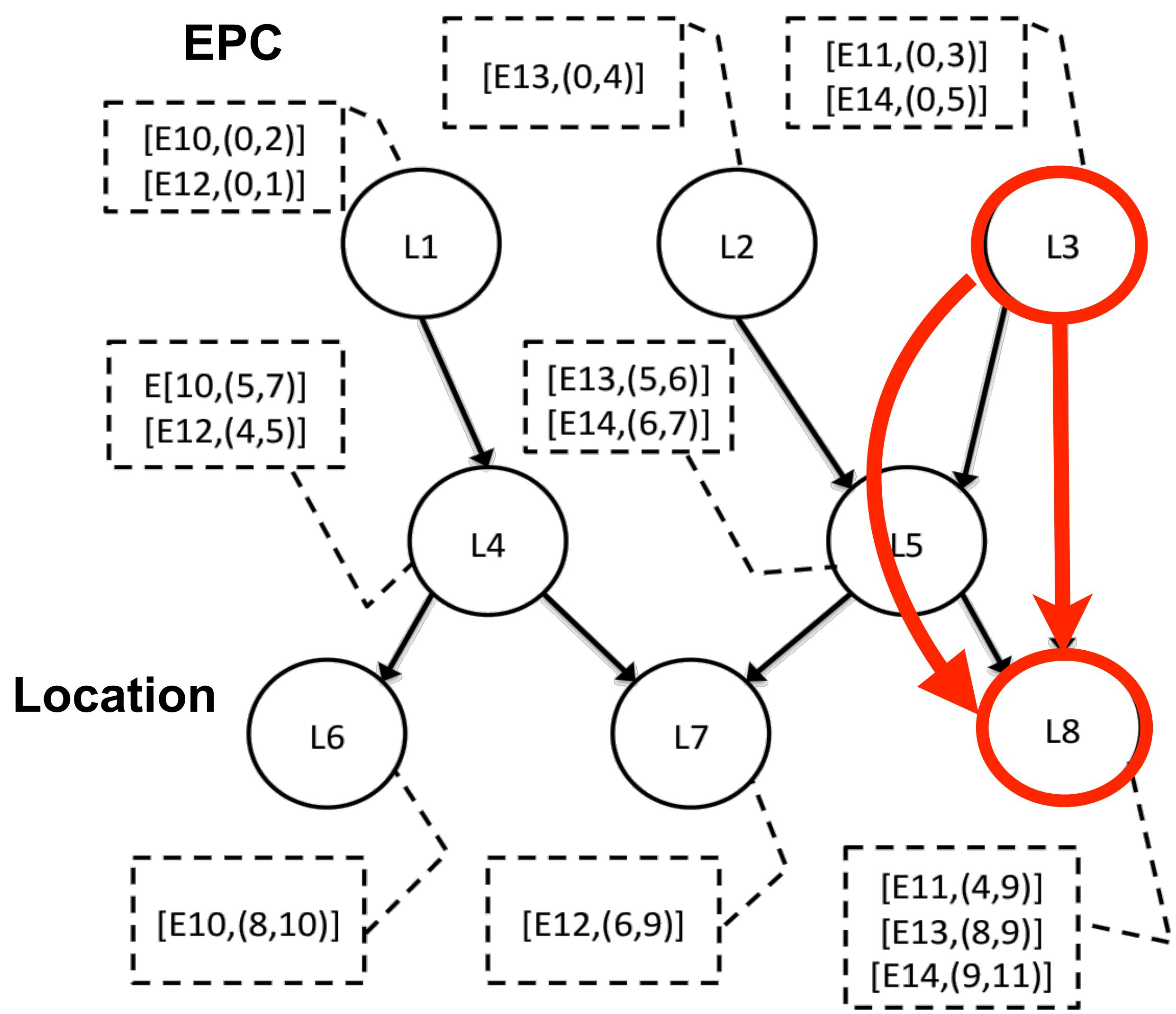


State-of-the-art drawbacks

(i) all RFID data representations are path-dependant: few flexibility to process more complex queries

(ii) only path queries over objects moving together

(iii) (un)efficient centralized ad-hoc storage model (and high consumption of resources)

Aim and Scope

- ☑ We provide a *general model* for supply chain management based on the first principles of linear algebra, in particular on tensorial calculus.
- ☑ It can facilitate both quick decentralized on-line processing, and centralized off-line massive business logic analysis, according to needs and requirements of supply chain actors.
- ☑ Such platform is implemented in a prototype tool: sIRA

RFID data: *a general model*

Let us define the set $\mathcal{E}$ of all EPCs, with $\mathcal{E}$ being finite. A *property* of an EPC is defined as an *application* $\pi_i : \mathcal{E} \rightarrow \Pi_i$, where Π_i represents a suitable property codomain.

A *supply chain (S)* is defined as the product set of all EPCs, and all the associated properties.

$$\mathcal{S} = \underbrace{\mathcal{E} \times \Pi_1 \times \ldots \times \Pi_{k-1}}_{\mathbb{N}^k} \times \underbrace{\Pi_k \times \ldots \times \Pi_{k+d}}_{\mathbb{U}} .$$

RFID data: *tensorial representation*

$$\mathcal{S} = \underbrace{\mathcal{E} \times \Pi_1 \times \ldots \times \Pi_{k-1}}_{\mathbb{N}^k} \times \underbrace{\Pi_k \times \ldots \times \Pi_{k+d}}_{\mathbb{U}} \,.$$

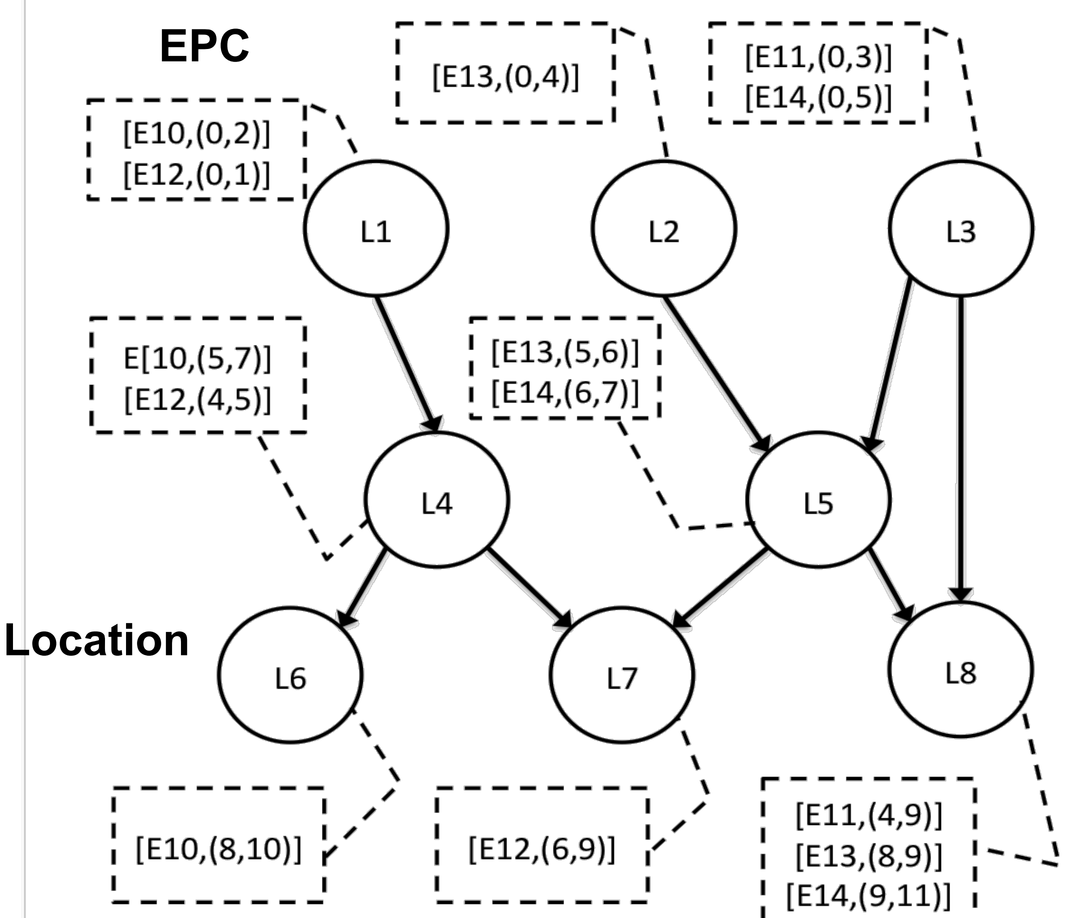
The tensorial representation of a *supply chain (S)* is a multilinear form

$$\Sigma : \mathbb{N}^k \longrightarrow \mathbb{U} \,.$$

Such mapping will therefore introduce a family of injective functions called *indexes*, defined as:

$$\mathrm{idx}_i : \Pi_i \longrightarrow \mathbb{N} \,, \qquad i = 1, \ldots, k-1 \,.$$

RFID data: *tensorial representation*



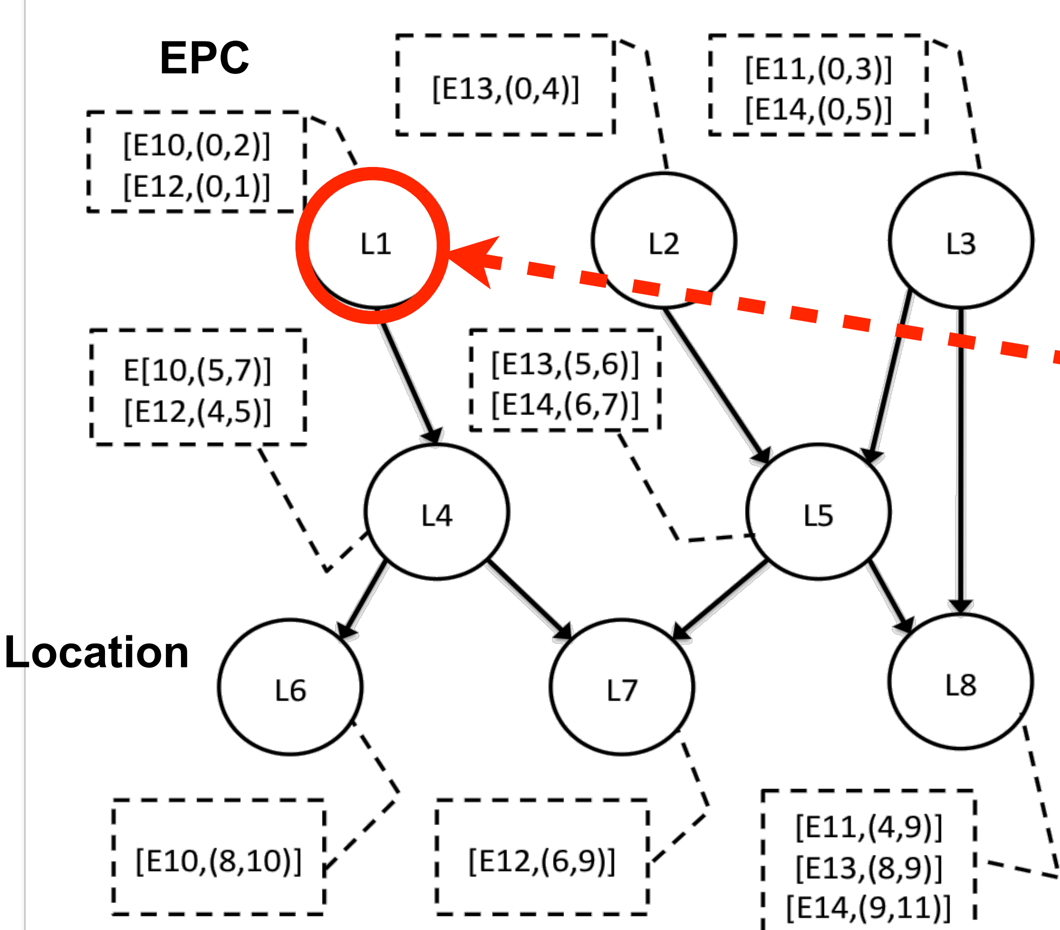
rank-k tensor: sparce matrix

	L1	L2	L3	L4	L5	L6	L7	L8
E10	$(0,2)$	·	·	$(5,7)$	·	$(8,10)$	·	·
E11	·	·	$(0,3)$	·	·	·	·	$(4,9)$
E12	$(0,1)$	·	·	$(4,5)$	·	·	$(6,9)$	·
E13	·	$(0,4)$	·	·	$(5,6)$	·	·	$(8,9)$
E14	·	·	$(0,5)$	·	$(6,7)$	·	·	$(9,11)$

$$\mathcal{E} = \{E10, E11, E12, E13, E14\}$$

$$\Pi_1 = \{L1, L2, L3, L4, L5, L6, L7, L8\}$$

RFID data: *tensorial representation*



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RFID data analysis: *tensorial application*

Tag Tracking: find the movement history for a given tag identifier *e* in $\mathcal{E}$.

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E14	·	·	(0, 5)	·	(6, 7)	·	·	(9, 11)

(1) $i = \mathrm{idx}(e)$

(2) $\delta_i, \ \text{with } |\delta_i| = |\mathcal{E}|$

(3) $r = M_{ij}\delta_i$.

(1) Es. *e* = E10. $i = \mathrm{idx}(E10) = 1$

(2) $\delta_1 = (1\ 0\ 0\ 0\ 0)^t$

(3) $r = M_{ij}\delta_1 = \{1\} \rightarrow (0,2),$
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Path Tracking: find the set of tag identifiers *e* that satisfy different conditions.

(1) $\forall z = \mathrm{idx}(L_k) \rightarrow \delta_z$

(2) $\forall z : r_z = M_{ij}\delta_j$

(3) $\widetilde{r_z} = \mathrm{map}(\mathrm{cond}_z, r_z)$

$\mathrm{cond}_z : \mathbb{N}^k \times \mathbb{U} \longrightarrow \mathbb{F}$

(4) $\bar{r} = \widetilde{r_1} \circ \ldots \circ \widetilde{r_n}$

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(1) Es. *L3[ti > 0]//L8[to - ti < 4]* $\mathrm{idx}(L3) = 3, \mathrm{idx}(L8) = 8$

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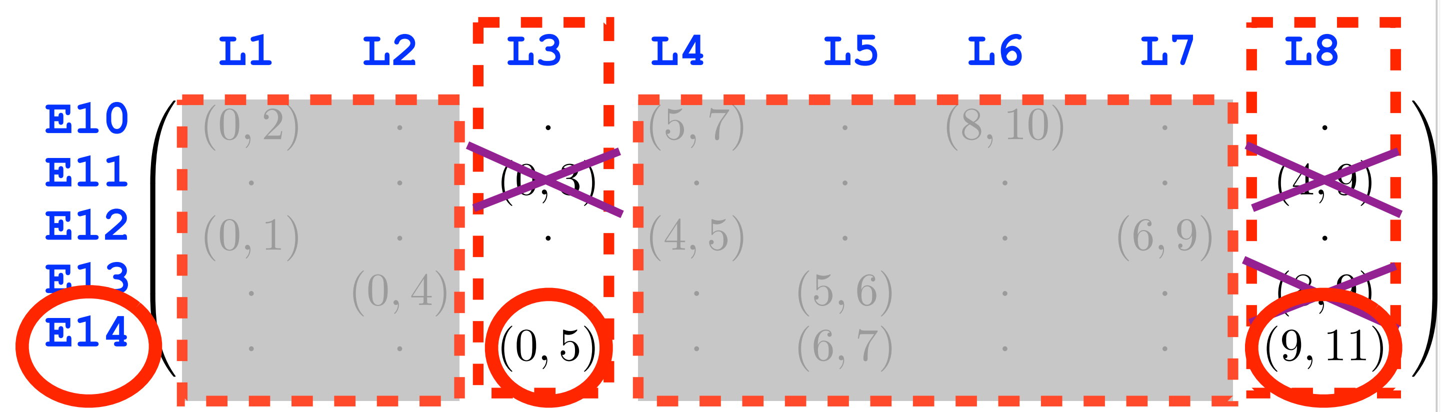

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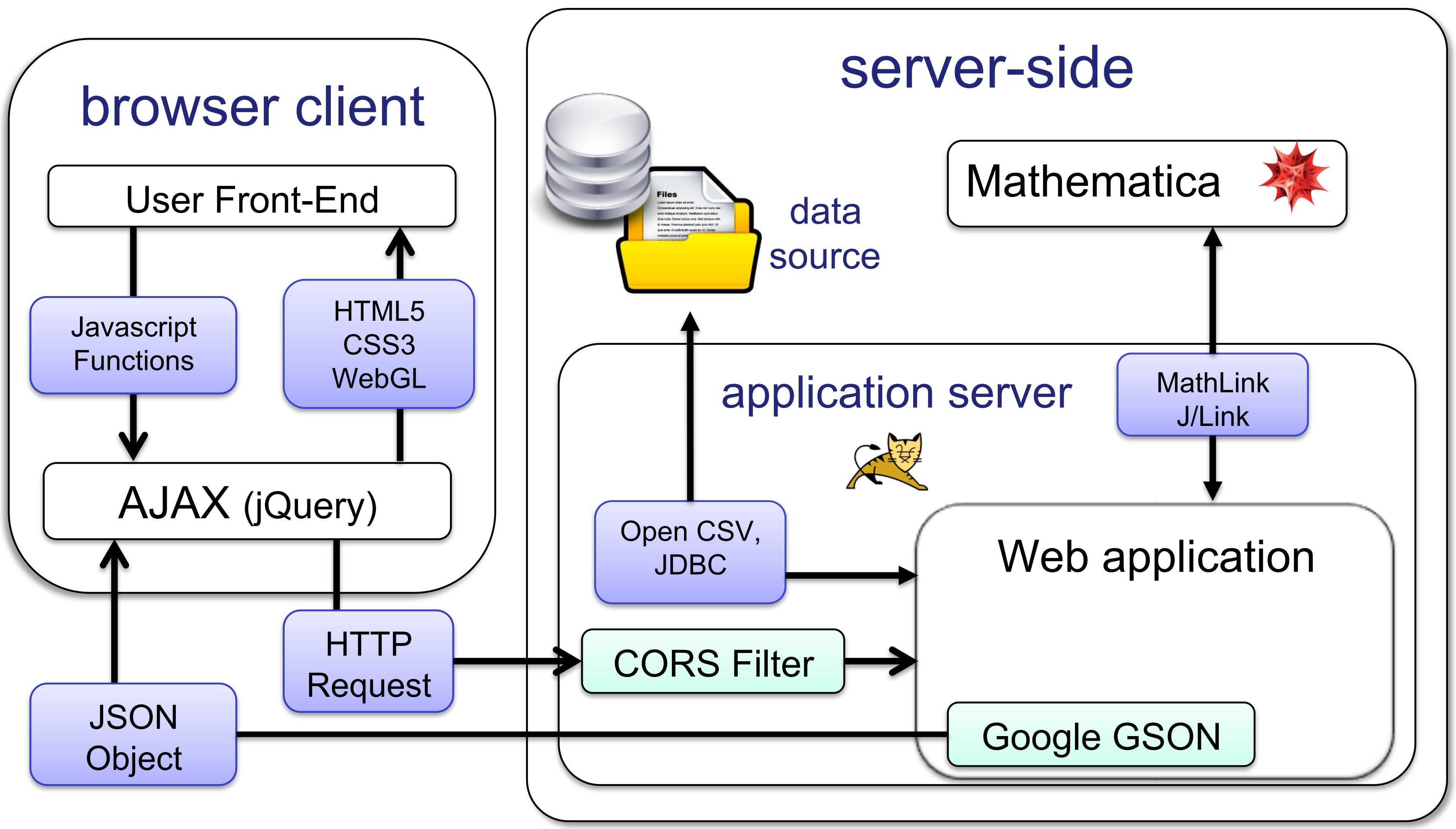
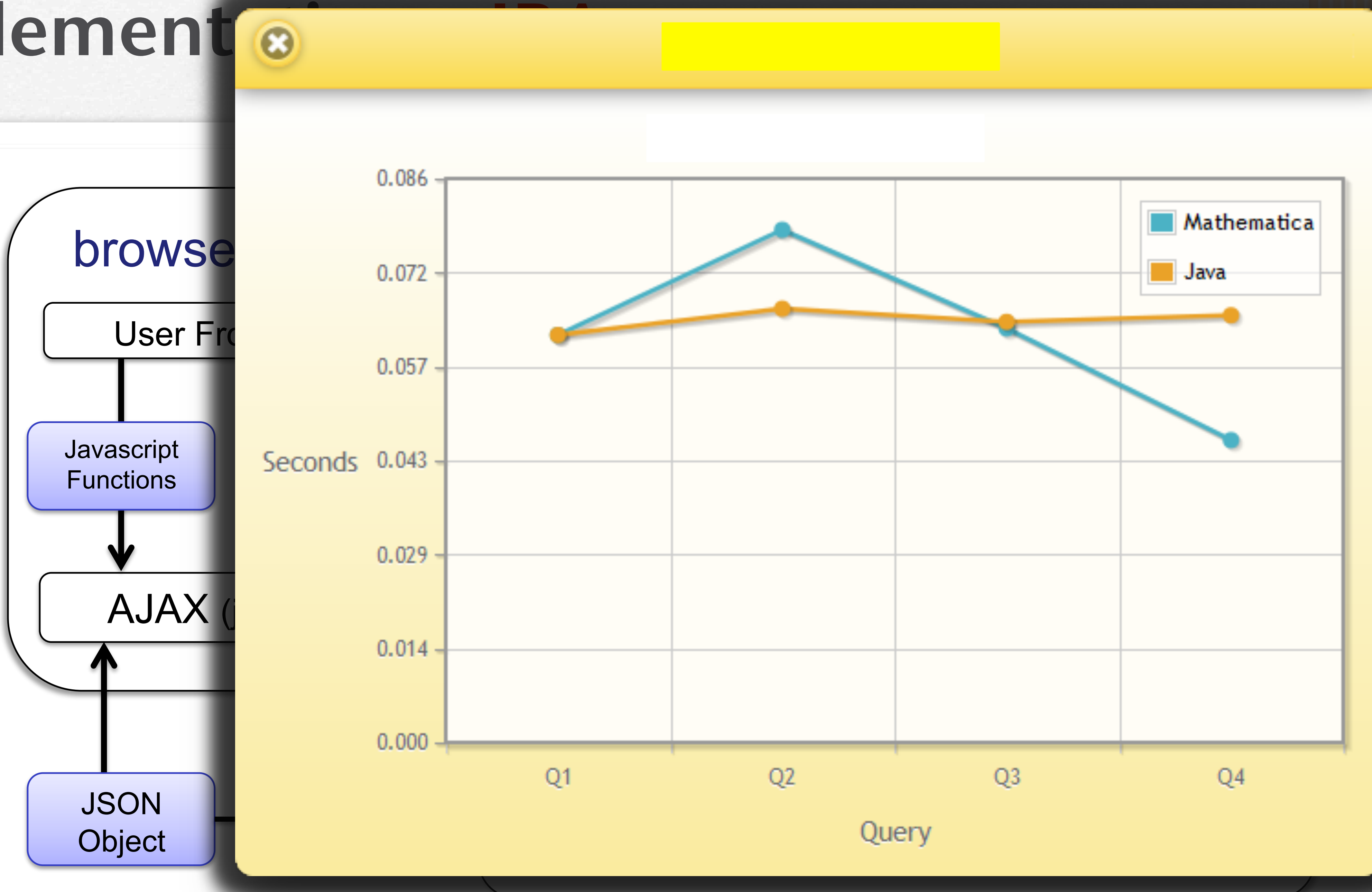
Implementation: sIRA



http://pamir.dia.uniroma3.it:8080/SimpleWebMathematica

Implement



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Comparative Study

We performed a series of experiments aimed at evaluating the performance of our approach: tracking, path-oriented retrieval and path-oriented aggregate queries (totally 12).

- ☑ Performance Analysis
- ☑ Memory Consumption

Our results have been compared to the ones from the approach of Lee at al. (SIGMOD 2008), tested against a generated synthetic RFID data in terms of stay records: 10^5, 10^6 and 10^7 stay records.

In the following, we will denote our tensorial approach T, while the proposed one by Lee et al. with P.

PATH_TABLE

PATH_ID	ELEMENT_ENC	ORDER_ENC

TAG_TABLE

TAG_ID	PATH_ID	START	END	TYPE

TIME_TABLE

START	END	LOC	START_TIME	END_TIME

INFO_TABLE

TYPE	PRODUCT_NAME	MANUFACTURER	PRICE

- ***A logical organization***
  - **query templates** for tracking queries and **path oriented queries** to analyze the supply chain
  - an **effective path encoding scheme** to encode the flow information for products

C.-H. Lee, C.-W. Chung:

Efficient storage scheme and query processing for supply chain management using RFID.

SIGMOD 2008

Data Loading

data	sec	MB
10^7	113	1450
$5 \cdot 10^6$	57	898
10^6	11	184
$5 \cdot 10^5$	8.2	92
10^5	0.9	13

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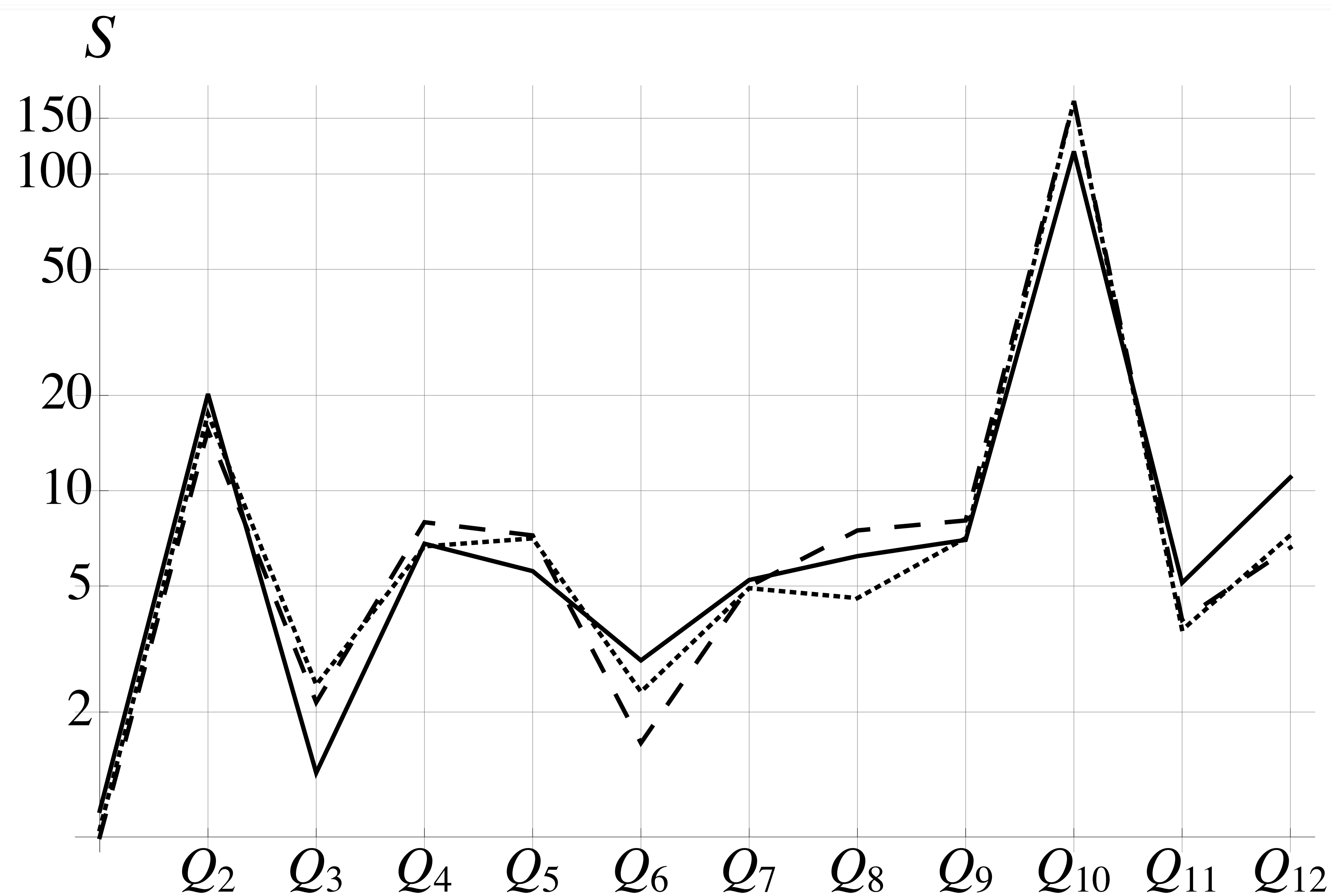
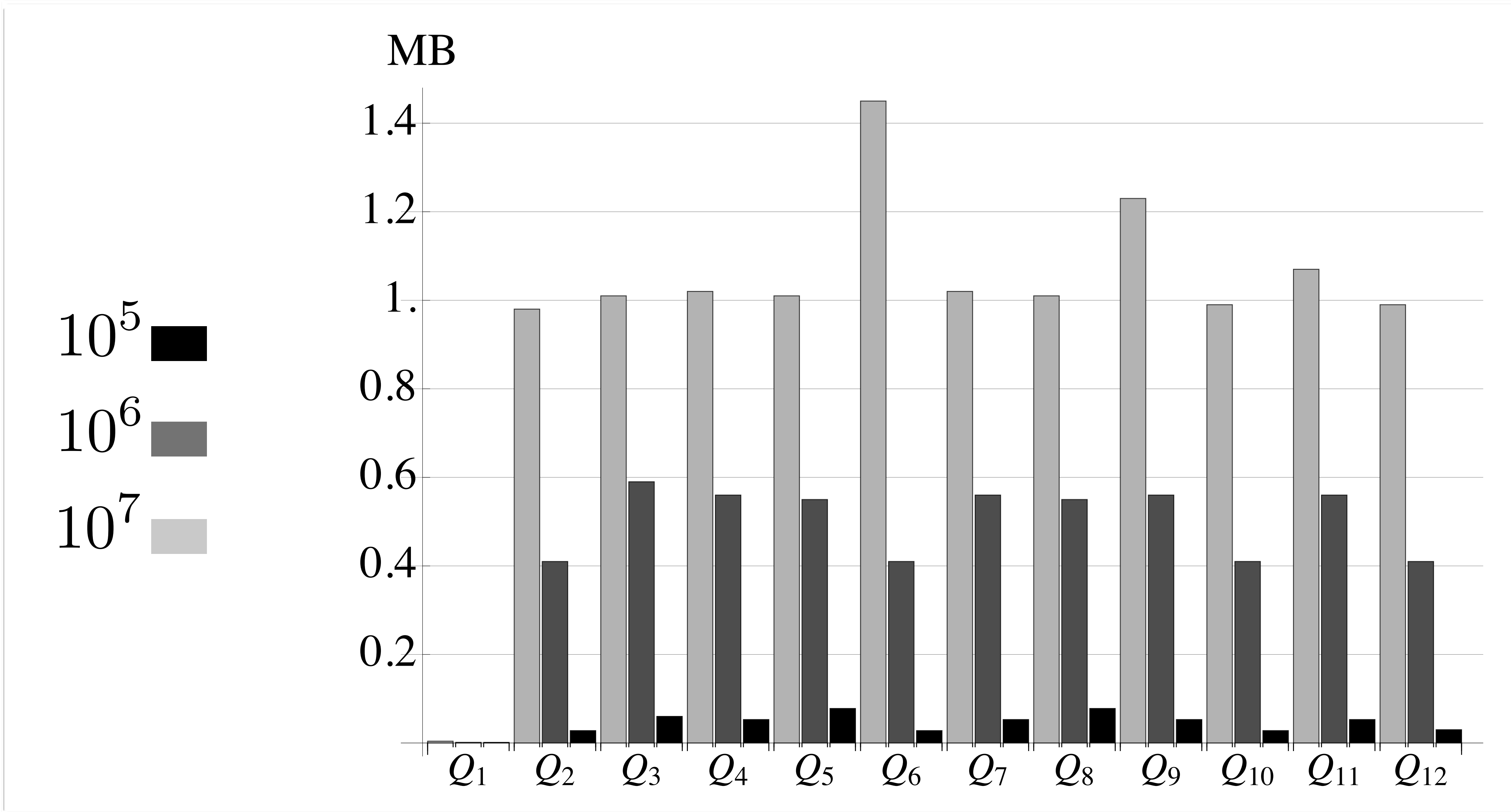
Performance (speed-up)



$$S = \frac{t_P}{t_T}$$

Memory Consumption



Conclusion and Future Work

- We have presented an abstract algebraic framework for the efficient analysis of RFID data in supply chain management
- Our approach leverages tensorial calculus, proposing a general model that exhibits a great flexibility with multidimensional queries, at diverse granularity and complexity levels.
- For future developments we are investigating the introduction of reasoning capabilities, along with a thorough deployment in highly distributed Grid environments.

THANKS FOR THE ATTENTION

... TO BE CONTINUED