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Stratification-based Criteria for Checking Chase Termination

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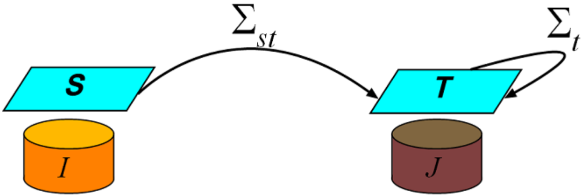
Data Exchange [FaginKMP05]

Schema Mapping $\mathcal{M} = (\mathbf{S}, \mathbf{T}, \Sigma_{st}, \Sigma_t)$



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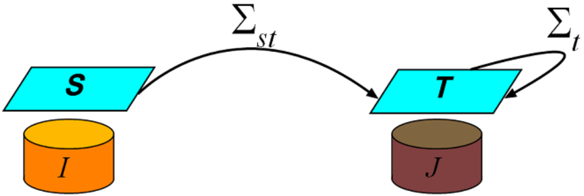

- Given a source instance I, a *solution for I under $\mathcal{M}$* is a target instance J s.t. $(I, J) \models \Sigma_{st}$ and $J \models \Sigma_t$.

Data Exchange - Universal solutions

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Let $\mathcal{M}$ be specified by the s-t TGD

$$\forall x \forall y (E(x, y) \rightarrow \exists z (F(x, z) \land F(z, y)).$$

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- Intuitively, universal solutions are the **most general solutions** in data exchange.

Data Exchange - Universal solutions

Theorem

A universal solution can be computed using the ***chase algorithm****.*

Chase Algorithm

- Add tuples to the database and set the value of nulls to satisfy integrity constraints.

Example

- $D = \{N(a), S(a)\}$

$$\Sigma = \begin{array}{l} \forall x\ [\ N(x) \rightarrow \exists y\ E(x,y)\ ] \\ \forall (x,y)\ [\ S(x) \wedge E(x,y) \rightarrow N(y)\ ] \end{array}$$

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- Other applications: query containment & optimization, data integration, consistent query answering,. . .

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- Chase termination: undecidable problem (Deutsch et al. [2008])
- Individuate sufficient chase termination conditions

Weak Acyclicity [FaginKMP05]

(Dependency Graph)

- represent the flow of values
- nodes are positions
- special edges $\stackrel{*}{\rightarrow}$ indicates the generation of new null values

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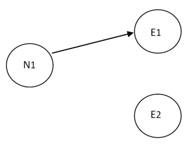

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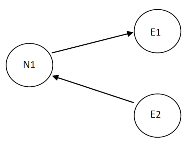

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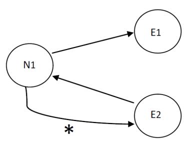
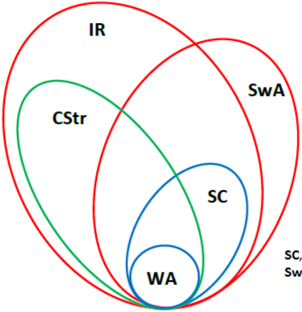


$\mathcal{WA}$, $\mathcal{SC}$, $\mathcal{CS}tr$, $\mathcal{IR}$ and $\mathcal{S}w\mathcal{A}$ Relationships



SC,CStr,IR [MeierSL09]
SwA [Marnette09]

Contributions

- Introduction of new chase termination criteria
  - *WA*-stratification (*SC*-stratification and *SwA*-stratification)
  - Local Stratification

Chase Termination Criteria

Stratification [DeutschNR08]

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- *Precedence relation*

  $r_1 \prec r_2$ if $\exists K_1$ s.t.

  1) $K_1 \models r_2 \wedge K_1 \xrightarrow{r_1} K_2$,

  2) $K_2 \not\models r_2$.

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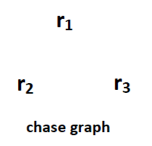
- Build the *chase graph*
- Partition in strongly connected components

Example: Stratification

Example

$$\Sigma = \begin{array}{ll} r_1: & R(x) \rightarrow \exists y\, T(x, y) \\ r_2: & R(x) \rightarrow T(x, x) \\ r_3: & T(x, y) \wedge T(x, x) \rightarrow R(y) \end{array}$$



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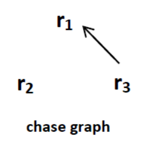

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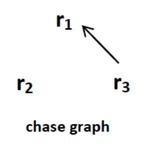

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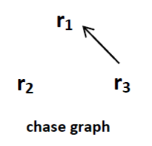

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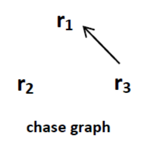

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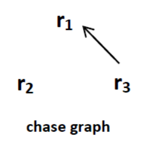

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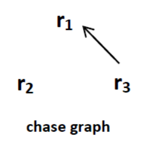

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  - if $K_1 = \{R(a), T(a,a)\}$ then $K_1 \models \Sigma$ (K_1 universal solution)

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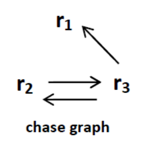

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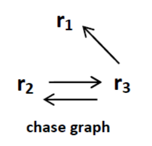

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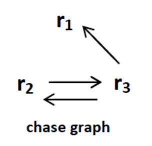

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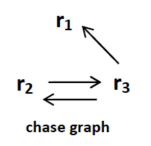

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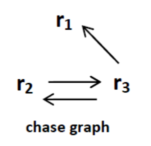

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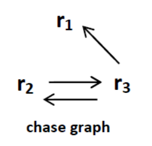

Σ is stratified

- chase sequence $r_1 \rightarrow r_2 \rightarrow r_3 \rightarrow \ldots$ is not terminating

  $D = \{R(a)\}$

  $D \xrightarrow{r_1,h_1} D_1 = \{R(a), T(a,\eta_1)\}$

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Example: Stratification

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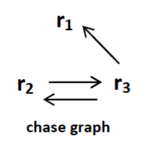

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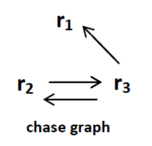

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Stratification

- Variant of stratification: C-Stratification

Stratification

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- Limited because uses a weaker form of chase

WA-stratification [Greco,Spezzano,Trubitsyna VLDB2011]

- Builds a *firing graph* $\Gamma(\Sigma) = (\Sigma, E)$ representing how constraints fire each other.
- $(r_1, r_2) \in E$ if $r_1 < r_2$ (firing r_1 can cause r_2 to fire)
- $r_1 < r_2$ if:

  1) $K_1 \xrightarrow{r_1} K_2$,

  2) $K_1 \cup S \models r_2$,

  3) $K_2 \cup S \not\models r_2$ and

  4) $Null(S) \cap (Null(K_2) - Null(K_1)) = \emptyset$.

- $r_1 \prec r_2$ if:

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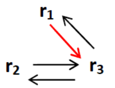
- $K_1 = \{R(a)\}$ and $K_2 = \{R(a), T(a, \eta_1)\}$
- $S = \{T(a,a)\}$
- $r_3: \; T(a, \eta_1) \wedge T(a,a) \rightarrow R(\eta_1)$
- r_3 is fired by r_1, then we have $r_1 < r_3$



firing graph

WA-stratification [Greco,Spezzano,Trubitsyna VLDB2011]

(WA-Stratification)

We say that Σ is *WA-stratified* (*WAStr*) iff the constraints in every nontrivial strongly connected component of the *firing graph* $\Gamma(\Sigma) = (\Sigma, \{(r_1, r_2) | r_1 < r_2\})$ are weakly acyclic.

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Proposition

$\mathcal{C}\mathcal{S}tr \subsetneq \mathcal{W}\mathcal{A}\mathcal{S}tr$ and $\mathcal{S}\mathcal{C} \nparallel \mathcal{W}\mathcal{A}\mathcal{S}tr$.

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Proposition

$\mathcal{CS}tr \subsetneq \mathcal{WAS}tr$ and $\mathcal{SC} \nparallel \mathcal{WAS}tr$.

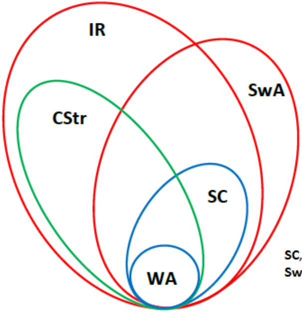
Example

$$\Sigma = \{r : T(x, y) \wedge T(y, x) \rightarrow \exists z\ T(y, z) \wedge T(z, x)\}$$

is *WA*-stratified, but not c-stratified.

SC-stratification and *SwA*-stratification

[Greco,Spezzano,Trubitsyna VLDB2011]



SC,CStr,IR [MeierSL09]
SwA [Marnette09]

SC-stratification and *SwA*-stratification

[Greco,Spezzano,Trubitsyna VLDB2011]

Definition

Given a set of TDGs Σ, we say that Σ is

- *SC-stratified* (*SCStr*) if the constraints in every nontrivial strongly connected component of the firing graph $\Gamma(\Sigma)$ are safe.
- *SwA-stratified* (*SwAStr*) if the constraints in every nontrivial strongly connected component of the firing graph $\Gamma(\Sigma)$ are super-weak acyclic.

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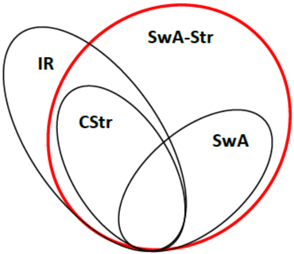
Proposition

Let Σ be a set of WA-stratified (resp. SC-stratified, SwA-stratified) TGDs and let D be a database. Then, the length of every chase sequence of Σ over D is polynomial in the size of D.

SwA-stratification [Greco,Spezzano,Trubitsyna VLDB2011]

Theorem

1. $\mathcal{WAS}tr \subsetneq \mathcal{SCS}tr \subsetneq \mathcal{S}w\mathcal{AS}tr$,
2. for $C \in \{WA, SC, SwA\}$, $\mathcal{C} \subsetneq \mathcal{C}\text{-}\mathcal{S}tr$ and
3. $\mathcal{IR} \nparallel \mathcal{S}w\mathcal{AS}tr$.



Local Stratification [Greco,Spezzano,Trubitsyna VLDB2011]

New chase termination condition: **Local Stratification (LS)**

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New chase termination condition: **Local Stratification (LS)**

- Combine $<$-relation of $\mathcal{WA}$-$\mathcal{S}tr$, with *SwA* criterion,
- ensures the termination of all chase sequences in polynomial data complexity,
- $\mathcal{S}w\mathcal{A}$-$\mathcal{S}tr \subsetneq \mathcal{LS}$ and $\mathcal{IR} \subsetneq \mathcal{LS}$.

Example: Super-weak Acyclicity [Marnette09]

Analyzes nulls propagation through constraints

Example

$$\Sigma = \begin{array}{ll} r_1: & N(x) \rightarrow \exists y\, \exists z\, E(x,y) \wedge S(z,y) \\ r_2: & E(x_1,y_1) \wedge S(x_1,y_1) \rightarrow N(y_1) \\ r_3: & E(x_2,y_2) \rightarrow E(y_2,x_2) \end{array}$$

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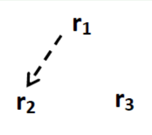

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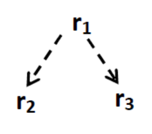

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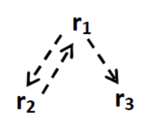

$$Move(r_1, y) = \{p_3, p_5, p_{10}, p_{13}, {\color{red}{p_2}}\}$$

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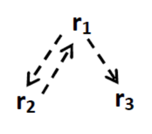

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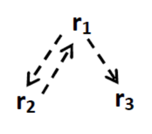

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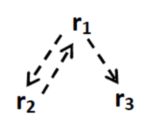

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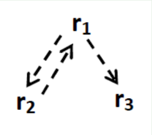

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The relation $\rightsquigarrow$ is cyclic and Σ is not in SwA

Local Stratification [Greco,Spezzano,Trubitsyna VLDB2011]

IDEA: testing the firing relation $<$ from *WA*-stratification in the *MOVE* construction

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$MOVE(r_1, y) = \{p_3, p_5\}$

$p_{10} \notin MOVE(r_1, y)$ as $r_1 \not< r_2$

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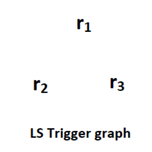

Conclusions

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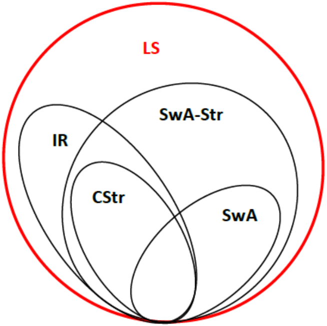
- We proposed new sufficient chase termination criteria, such as *WA*(*SC*,*SwA*)-Stratification and Local Stratification



Thank you!

Any question?