An Extension of Datalog for Graph Queries

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- and define a new type of predicates called Multi-Occuring Predicates.

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- Example about friends that will come to the party.

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- It is not monotone and requires stratified negation.

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$$\texttt{tref}(\texttt{Author}) : \texttt{N} \leftarrow \quad \texttt{N} : [\texttt{author}(\texttt{Author}, \texttt{Pno}), \texttt{ref}(\texttt{Pno})].$$

$$total\_ref(A) = \sum_{Pno \in paper(A)} reference(Pno)$$

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- Semantics by using Datalog.

$$\mathtt{ref("Bob2012", J)} \leftarrow \quad \mathtt{lessthan(J, 6).}$$
$$\mathtt{tref(Author, N)} \leftarrow \quad \mathtt{N : [author(Author, Pno), ref(Pno, J)].}$$

$$\mathtt{lessthan(1, K)} \leftarrow \quad \mathtt{K} \geq \mathtt{1.}$$
$$\mathtt{lessthan(J1, K)} \leftarrow \quad \mathtt{lessthan(J, K), K > J, J1 = J + 1.}$$

Applications

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Number of paths between two nodes in a Direct Acyclic Graph (DAG).

$$\begin{aligned} \mathtt{path}(\mathtt{X},\mathtt{Y}) : 1 &\leftarrow \quad \mathtt{edge}(\mathtt{X},\mathtt{Y}) \\ \mathtt{path}(\mathtt{X},\mathtt{Y}) : \mathtt{K} &\leftarrow \quad \mathtt{K}\!:\![\mathtt{path}(\mathtt{X},\mathtt{Z}),\mathtt{path}(\mathtt{Z},\mathtt{Y})]. \end{aligned}$$

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Number of paths between two nodes in a Direct Acyclic Graph (DAG)

$$n\_path(X, Y) = \sum_{Z \neq X, Y} n\_path(X, Z) \times n\_path(Z, Y)$$

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Example

Reachability in a directed hyper graph.
A hyperedge $(\{a, b\}, c)$ is represented by the following facts:

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- *........*

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- the percentage of neighbors that changed behavior.

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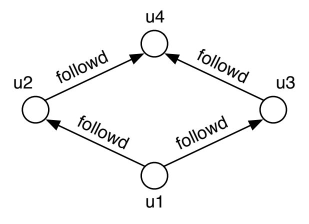
$$\begin{aligned}
\texttt{coeff}(\texttt{X},\texttt{C}) \leftarrow \quad & \texttt{K2} =![\texttt{followd}(\texttt{Y},\texttt{X})], \texttt{bc}(\texttt{X},\texttt{V1}),\\
& \texttt{g}(\texttt{K2},\texttt{V3}), \texttt{C} = \texttt{V1} * \texttt{V3}/\texttt{K2}.\\
\texttt{b}(\texttt{X}) \leftarrow \quad & \texttt{source}(\texttt{X}).\\
\texttt{b}(\texttt{X}) \leftarrow \quad & \texttt{coeff}(\texttt{X},\texttt{C}), \texttt{K} \geq 1/\texttt{C}, \texttt{K} : [\texttt{followd}(\texttt{Y},\texttt{X}), \texttt{b}(\texttt{Y})].
\end{aligned}$$



$\texttt{follwd}(u_1, u_2)$.	$\texttt{bc}(u_1, 1)$.	$\texttt{g}(1, 1.2)$.
$\texttt{follwd}(u_1, u_3)$.	$\texttt{bc}(u_2, 0.9)$.	$\texttt{g}(2, 2.3)$.
$\texttt{follwd}(u_2, u_4)$.	$\texttt{bc}(u_3, 0.5)$.	
$\texttt{follwd}(u_3, u_4)$.	$\texttt{bc}(u_4, 1)$.	
$\texttt{source}(u_1)$.		

Jackson-Yariv Diffusion Model

$$\begin{aligned}
\mathtt{coeff}(\mathtt{X},\mathtt{C}) \leftarrow \quad & \mathtt{K2} = ![\mathtt{followd}(\mathtt{Y},\mathtt{X})], \mathtt{bc}(\mathtt{X},\mathtt{V1}),\\
& \mathtt{g}(\mathtt{K2},\mathtt{V3}), \mathtt{C} = \mathtt{V1} * \mathtt{V3}/\mathtt{K2}.\\
\mathtt{b}(\mathtt{X}) \leftarrow \quad & \mathtt{source}(\mathtt{X}).\\
\mathtt{b}(\mathtt{X}) \leftarrow \quad & \mathtt{coeff}(\mathtt{X},\mathtt{C}), \mathtt{K} \geq 1/\mathtt{C}, \mathtt{K} : [\mathtt{followd}(\mathtt{Y},\mathtt{X}), \mathtt{b}(\mathtt{Y})].
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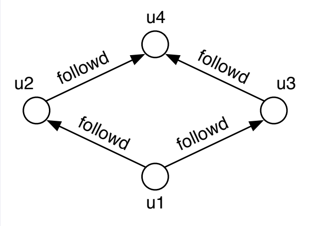

$$\begin{array}{lll}
\mathtt{follwd}(\mathtt{u}_1,\mathtt{u}_2). & \mathtt{bc}(\mathtt{u}_1,1). & \mathtt{g}(1,1.2).\\
\mathtt{follwd}(\mathtt{u}_1,\mathtt{u}_3). & \mathtt{bc}(\mathtt{u}_2,0.9). & \mathtt{g}(2,2.3).\\
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\mathtt{follwd}(\mathtt{u}_3,\mathtt{u}_4). & \mathtt{bc}(\mathtt{u}_4,1). & \\
\mathtt{source}(\mathtt{u}_1). & &
\end{array}$$

the program derives the following atoms:

$$\begin{array}{lll}
\mathtt{coeff}(\mathtt{u}_2,1.08), & \mathtt{coeff}(\mathtt{u}_3,0.6), & \mathtt{coeff}(\mathtt{u}_4,1.15),\\
\mathtt{b}(\mathtt{u}_1), & \mathtt{b}(\mathtt{u}_2), & \mathtt{b}(\mathtt{u}_4).
\end{array}$$

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- Used for Page Rank.

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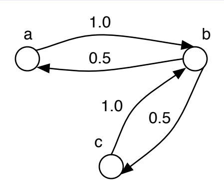

$$\mathtt{p\_st}(\mathtt{X}) : \mathtt{K} \leftarrow \quad \mathtt{K} : [\mathtt{p\_st}(\mathtt{Y}), \mathtt{w\_mat}(\mathtt{Y}, \mathtt{X})].$$

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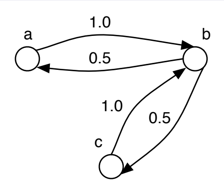

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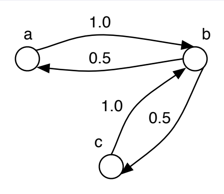

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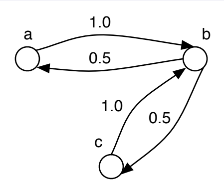

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- Final-FS goal: its semantics is defined by using the negation. The stratified fixpoint for programs with stratified negation is used.
- Multiplicity predicate: we only store the maximum value of multiplicity.

Conclusion and Future Work

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- Recent works show how it is possible to use parallel paradigms (Map Reduce) to improve Datalog's performances.
  - Datalog^{FS} and in particular the multiplicity predicates can be also combined with such technologies.

Thank you!
Any question?

Compact way

Compact way of expressing counting goals:

- in Datalog^{FS}

$$\texttt{sixsubs}(\texttt{X}) \leftarrow \texttt{detective}(\texttt{X}), 6 : [\texttt{superior}(\texttt{X}, \texttt{Y})].$$

- in Datalog

$$\begin{aligned}\texttt{sixsubs}(\texttt{X}) \leftarrow\ & \texttt{detective}(\texttt{X}), \texttt{superior}(\texttt{X}, \texttt{Y1}), \\ & \texttt{superior}(\texttt{X}, \texttt{Y2}), \texttt{superior}(\texttt{X}, \texttt{Y3}), \\ & \texttt{superior}(\texttt{X}, \texttt{Y4}), \texttt{superior}(\texttt{X}, \texttt{Y5}), \\ & \texttt{superior}(\texttt{X}, \texttt{Y6}), \texttt{Y1} \neq \texttt{Y2}, \texttt{Y1} \neq \texttt{Y3}, \\ & \texttt{Y1} \neq \texttt{Y4}, \texttt{Y1} \neq \texttt{Y5}, \texttt{Y1} \neq \texttt{Y6}, \texttt{Y2} \neq \texttt{Y3}, \\ & \texttt{Y2} \neq \texttt{Y4}, \texttt{Y2} \neq \texttt{Y5}, \texttt{Y2} \neq \texttt{Y6}, \texttt{Y3} \neq \texttt{Y4}, \\ & \texttt{Y3} \neq \texttt{Y5}, \texttt{Y3} \neq \texttt{Y6}, \texttt{Y4} \neq \texttt{Y5}, \texttt{Y4} \neq \texttt{Y6}, \\ & \texttt{Y5} \neq \texttt{Y6}.\end{aligned}$$